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Malapportionment in the European Parliament of the EU-27: 10th Legislature 2024-2029, Population Census 2025

Malapportionment en el Parlamento Europeo de la UE-27: X Legislatura 2024-2029, Censo poblacional 2025

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Abstract
This study analyzes the representation of European Union citizens in the European Parliament during its 10th legislature (2024-2029). The analysis takes into account the official population census of 2025 and European Council Decision (EU) 2023/2061. Seven seat allocation methods (Hamilton, Adams, Dean, Hill, Webster, Jefferson, and Belga) are applied among the 27 EU-27 Member States. The seat allocations proposed by these methods are compared with the current allocation resulting from the application of the European Electoral System. The quality of the equitable distribution of power among the Member States is measured using 15 territorial representativeness metrics.

Keywords: malapportionment, European Union, European Parliament, European Council, EU-27, apportionment problem, disproportionality indices.

Resumen
Se analiza la representación de la ciudadanía de la Unión Europea en el Parlamento Europeo en su X Legislatura (2024-2029). Para el análisis, se han tenido en cuenta el censo poblacional oficial del año 2025 y la Decisión (UE) 2023/2061 del Consejo Europeo. Se aplican siete métodos de reparto de escaños (Hamilton, Adams, Dean, Hill, Webster, Jefferson y Belga) entre los 27 Estados miembros de la UE-27. Se comparan los repartos de escaños propuestos por estos métodos con el reparto actual, resultante de aplicar el Sistema Electoral Europeo. La calidad del reparto del poder de forma equitativa entre los Estados miembros se mide mediante 15 métricas de representatividad territorial.

Palabras clave: *malapportionment*, Unión Europea, Parlamento Europeo, Consejo Europeo, UE-27, reparto proporcional, índices de desproporcionalidad.

1. Preliminaries

The European Parliament directly represents the interests of the citizens of the European Union member states. Its main function is to exercise legislative and budgetary power jointly with the Council of the European Union.

Members of the European Parliament are elected by universal, direct, proportional, and secret suffrage in elections held by the citizens. Since 1979, these elections have been held every five years, and all member states recognized at the time of the election participate.

The last elections to the European Parliament were held between June 6 and 9, 2024, ushering in the tenth legislature, which replaced the ninth.

The current European Union (EU-27) is composed of 27 Member States: Germany, France, Italy, Spain, Poland, Romania, the Netherlands, Belgium, Czechia, Sweden, Portugal, Greece, Hungary, Austria, Bulgaria, Denmark, Finland,

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Slovakia, Ireland, Croatia, Lithuania, Slovenia, Latvia, Estonia, Cyprus, Luxembourg, and Malta.

The electoral systems of the European Union member states are not identical. In principle, they all follow proportional representation systems, although differences exist between them.

The so-called European Electoral System is based on common principles for the election and counting of votes, although it is open to further development by each state within certain rules. These basic principles are:

1. Universal suffrage (once the minimum voting age is reached).
2. Free and secret ballot by all citizens.
3. Direct vote for political parties.
4. Proportional allocation of seats based on the votes received by the political parties participating in the election.

Each state uses its own electoral system to allocate its assigned seats. Thus, Germany, Spain, Greece, France, and Portugal opted for a single national constituency in 2024, while countries like Ireland and Italy chose to divide their territory into several constituencies to elect their representatives.

Before the elections, a specific number of seats in the European Parliament were allocated to each Member State. For the elections to the Tenth Legislature (June 2024), the European Council established the composition of Parliament by Decision (EU) 2023/2061 of 22 September 2023 (European Council, 2023). The articles relevant to this paper are taken from that Decision.

Article 1:

"In the application of Article 14(2) TEU, the following principles shall be respected:

- *the total number of seats in the European Parliament shall not exceed 750 in number, plus the President*
- *the allocation of seats to Member States shall be degressively proportional with a minimum threshold of six seats and a maximum threshold of 96 seats per Member State, while reflecting as closely as possible the sizes of the respective populations of the Member States;*
- *degressive proportionality is defined as follows: the ratio between the population and the number of seats of each Member State before rounding up or down to the nearest whole number is to vary in relation to their respective populations in such a way that each Member of the European Parliament from a more populous Member State represents more citizens than each Member of the European Parliament from a less populous Member State and, conversely, that the larger the population of a Member State, the greater its entitlement to a large number of seats in the European Parliament;*
- *the allocation of seats in the European Parliament is to consider demographic developments in the Member States.*

Article 2:

The total population of the Member States is calculated by the Commission (Eurostat) on the basis of data provided by the Member States, in accordance with a method established by means of Regulation (EU) No 1260/2013 of the European Parliament and of the Council[2]*.*

Article 3:

The number of representatives in the European Parliament elected in each Member State for the 2024–2029 parliamentary term is set as follows: (see Table 1).

Table 1. Distribution of seats among the 27 Member States of the European Union (EU-27) within the European Parliament (Tenth Legislature 2024-2029. Composition: 720 seats[3]).

State	Seats	State	Seats	State	Seats
Germany	96	Portugal	21	Slovakia	14
France	81	Sweden	21	Croatia	12
Italy	76	Greece	21	Lithuania	11
Spain	61	Hungary	21	Slovenia	9
Poland	53	Austria	20	Latvia	9
Romania	33	Bulgaria	17	Estonia	7
Netherlands	31	Denmark	15	Cyprus	6
Belgium	22	Finland	15	Luxembourg	6
Czechia	21	Ireland	15	Malta	6

Article 4:

Sufficiently far in advance of the beginning of the 2029–2034 parliamentary term, and if possible by the end of 2027, the European Parliament shall submit to the European Council, in accordance with Article 14(2) TEU, a proposal for an updated allocation of seats in the European Parliament.".

[2] Regulation (EU) No 1260/2013 of the European Parliament and of the Council of 20 November 2013 on European demographic statistics (OJ L 330, 10.12.2013, p. 39).

[3] On September 15, 2023, the European Parliament approved the allocation proposed by the European Council with 515 votes in favor, 74 against and 44 abstentions.

Obviously, compliance with the principle of decreasing proportionality added to the minimum and maximum seat allocations to the Member States of the Union (Article 1 Decision (EU) 2023/2061) may result in a disproportionate distribution of power between the territories represented in the European Parliament.

In this context, the following questions arise:

p1. Cost of a seat in each EU-27 Member State (population/seat).
p2. States with overrepresentation in the European Parliament, 10th legislature.
p3. States with underrepresentation in the European Parliament, 10th legislature.
p4. Electoral bonuses for Member States in the European Parliament, 10th legislature.
p5. Representation of Member States in the European Parliament, 10th legislature.
p6. Relative electoral effort of Member States to obtain parliamentary representation.
p7. Inequality of parliamentary representation among Member States.
p8. Compliance with the principle of "one person, one vote" in the European Parliament.

This work is an extension and update of Bautista (2025a), in which we will try to answer the issues raised, using the official census of the European population of 2025, updated by Eurostat (2026).

The remainder of this work is structured as follows. Section 2 defines and illustrates the main metrics of partial territorial representativeness. Section 3 presents the most commonly used global indices of disproportionality in electoral systems. Section 4 is dedicated to models and methods of proportional representation used to allocate seats in representative chambers. Section 5 applies these methods and indices to the case of the European Parliament (10th Legislature: 2024-2029). Finally, Section 6 presents the partial and general conclusions of this work.

2. Metrics of partial territorial representativeness

Similar to Bautista (2025a, 2025b), to define the representativeness metrics we will use the following nomenclature:

Let be:

I	Set of Member States that constitute the Territory under study. Index of Member States: $i = 1,..,n$.
$p_i, \vec{p}$	Population registered in Member State $i \in I$: number of people affected by an electoral contest in $i \in I$. In vector form: $\vec{p} = (p_1, \ldots, p_n)$
P	Total population of the Territory $P = \sum_{i=1}^{n} p_i$. It is the total number of people affected by an electoral contest (EU-27).
$\pi_i, \vec{\pi}$	Proportion of the population of Member State $i \in I$: $\pi_i = p_i / P$ $(\forall i)$.In vector form: $\vec{\pi} = (\pi_1, \ldots, \pi_n)$.
H	Number of seats to be allocated among the Member States. This is linked to the size of the chamber of representatives (720 seats in the European Parliament in 2024).
$x_i, \vec{x}$	Number of seats allocated to Member State $i \in I$, depending on the allocation method. In vector form: $\vec{x} = (x_1, \ldots, x_n)$.
$\xi_i, \vec{\xi}$	Proportion of seats allocated to the Member State $i \in I$: $\xi_i = x_i / H$ $(\forall i)$, depending on the allocation method. In vector form: $\vec{\xi} = (\xi_1, \ldots, \xi_n)$
$\bar{c}$	Average electoral cost of the Territory: $\bar{c} = P/H$. It is the average number of people needed to obtain a seat in the electoral contest. The value $\bar{c}$ depends on the moment in which the electoral census is carried out: $\bar{c} = \bar{c}(t)$.

Under such conditions, the following partial representation indices are defined:

IP1. *Proportional territorial quota of a State*

Given the chamber size H and the population vector $\vec{p} = (p_1, \ldots, p_n)$, we call the territorial quota q_i of State $i \in I$ the number of seats that corresponds proportionally to State $i \in I$ based on its population; that is:

$$q_i = \frac{p_i}{P} H = \pi_i H (\forall i \in I) : \sum_{i=1}^{n} q_i = H; \ \vec{q} = (q_1, \ldots, q_n) \quad (1)$$

Territorial quotas are rational numbers that represent the ideal allocation of seats.

IP2. *Electoral bonus of a State*

Given the chamber size H, the vector of assigned seats $\vec{x} = (x_1, \ldots, x_n)$ and the vector of populations $\vec{p} = (p_1, \ldots, p_n)$, the electoral bonus s_i of State $i \in I$ is defined as the difference between the proportion of parliamentary seats assigned to State $i \in I$ and the proportion of inhabitants (within the EU-27) that corresponds to State $i \in I$; that is:

$$s_i \equiv \xi_i - \pi_i = \frac{x_i}{H} - \frac{p_i}{P} (\forall i \in I); \ \vec{s} = (s_1, \ldots, s_n) \quad (2)$$

When $s_i = 0$, State $i \in I$ is well represented in Parliament, while $s_i > 0$ implies that State $i \in I$ is overrepresented and if $s_i < 0$, it is underrepresented.

IP3. *Population representation index of a State*

Given the chamber size H, the vector of assigned seats $\vec{x} = (x_1, \ldots, x_n)$ and the population vector $\vec{p} = (p_1, \ldots, p_n)$, the population representation index r_i of State $i \in I$ is the ratio

between the proportion of seats of State $i \in I$ and its population proportion:

$$r_i \equiv \frac{\xi_i}{\pi_i} = \frac{x_i / H}{p_i / P} = \frac{x_i}{q_i} (\forall i \in I);\ \vec{r} = (r_1,\ldots,r_n) \quad (3)$$

If $r_i = 1$, State $i \in I$ is well represented, while if $r_i > 1$, State $i \in I$ is overrepresented, and if $r_i < 1$, $i \in I$ is underrepresented. The ratio r_i is also called the advantage ratio of the constituency or State $i \in I$ (Simón, 2009).

IP4. *Absolute territorial electoral cost of a State*

Given the chamber size H, the vector of assigned seats $\vec{x} = (x_1, \ldots, x_n)$ and the vector of populations $\vec{p} = (p_1, \ldots, p_n)$, the absolute territorial electoral cost c_i of State $i \in I$ is defined as the average number of people from State $i \in I$ represented by a seat within Parliament:

$$c_i = \frac{p_i}{x_i} (\forall i \in I) : \sum\nolimits_{i=1}^{n} x_i = H;\ \vec{c} = (c_1,\ldots,c_n) \quad (4)$$

Therefore, the cost c_i represents the number of inhabitants represented by each seat assigned to State $i \in I$.

IP5. *Relative territorial electoral cost of a State*

Given the vector of absolute electoral costs $\vec{c} = (c_1, \ldots, c_n)$ of a territory and the corresponding average electoral cost: $\bar{c} = P/H$, the relative territorial electoral cost cr_i of State $i \in I$ is defined as the ratio between the territorial cost of State $i \in I$ and the average electoral cost $\bar{c}$ of the Territory (European Union). That is:

$$cr_i = \frac{c_i}{\bar{c}} = \frac{p_i / x_i}{P / H} = \frac{p_i H}{x_i P} = \frac{q_i}{x_i} = \frac{1}{r_i} (\forall i \in I);\ \vec{cr} = (cr_1,\ldots,cr_n) \quad (5)$$

The relative cost cr_i determines the cost to State $i \in I$ of obtaining a seat relative to the average cost for the Territory. The function cr_i is the inverse of the population representation index r_i (advantage ratio) of State $i \in I$. Under conditions of perfect proportional allocation, the relative costs of the states must be equal to 1.

IP6. *Relative territorial effort of a State*

Given the chamber size H, the vector of assigned seats $\vec{x} = (x_1, \ldots, x_n)$ and the vector of populations $\vec{p} = (p_1, \ldots, p_n)$, the relative territorial effort e_i of State $i \in I$ is defined as the ratio between the territorial cost of State $i \in I$ and the minimum territorial cost. That is:

$$e_i = \frac{c_i}{c_{\min}} (\forall i \in I) : c_{\min} = \min_{i' \in I} c_{i'};\ \vec{e} = (e_1,\ldots,e_n) \quad (6)$$

Therefore, the effort e_i represents the cost of a seat to State $i \in I$ compared to the cost of a seat to the State that incurs the least cost.

Applying the territorial metrics IP1 to IP6 to the official census of the European Union in 2025 (Eurostat, 2026) yields the results shown in Table 2.

Based on Table 2 we can state:

1. The number of seats allocated (x_i) to EU member states differs significantly from their respective quotas (q_i), which represent the theoretical number of seats that each state would proportionally receive based on its population. For example, Germany receives 96 seats in the European Parliament with a quota of 133.53 seats according to its population, while Cyprus, Luxembourg, and Malta each receive 6 seats with quotas of 1.57, 1.09, and 0.92 seats, respectively.
2. The European Parliament has five underrepresented member states: Germany, France, Italy, Spain, and Poland; the remaining 22 EU-27 Member States are overrepresented. For example, Germany and France receive 37.53 and 29.05 fewer seats, respectively, while Bulgaria and Lithuania receive 6.72 and 6.38 more seats, respectively, than their quotas would allow. These and other imbalances result in a transfer of 107.55 seats from the five underrepresented member states to the remaining 22.
3. Considering electoral bonuses (s_i), Germany and France are the most disadvantaged states, with bonuses of -5.21% and -4.04%, respectively. On the other hand, Bulgaria is the most benefited state with a premium of 0.93%, followed by Lithuania with a premium of 0.89%.
4. Considering the representation index (r_i), Germany, France, and Spain are the three most disadvantaged states with respective indices of 0.72, 0.74, and 0.78. The three most benefited states are Malta (6.54), Luxembourg (5.51), and Cyprus (3.82).
5. The absolute territorial costs (c_i) for 2025 vary considerably among the 27 EU member states, with Germany having a maximum cost of 870,595 inhabitants per seat and Malta a minimum cost of 95,708 inhabitants per seat.
6. The relative territorial costs (cr_i) fall within the range [0.15; 1.39]. Germany experiences a relative cost increase of 39%, while Malta has an 85% savings in electoral costs.
7. Germany's relative effort (e_i) compared to Malta is 9.10, France's is 8.89, and Spain's is 8.42. This means that one vote in Malta is worth more than 9 votes in Germany and more than 8 votes in France or Spain, if the voter-to-population ratio is similar in these countries.
8. Similarly, one vote in Luxembourg is worth the same as 7.66 votes in Germany, 7.48 in France, and 7.09 in Spain.

Table 2. Population 2025 (p_i), number of seats (x_i), territorial quota (q_i), electoral bonus (s_i), representation index (r_i), relative territorial cost (cr_i), relative territorial effort (e_i) and absolute territorial cost (c_i) of the EU-27 European Union States. Population 2025 (p_i), Eurostat (2026)

EU-27/2025	p_i	x_i	q_i	s_i (%)	r_i	cr_i	e_i	c_i
Germany	83,577,140	96	133.53	-5.21	0.72	1.39	9.10	870,595
France	68,882,600	81	110.05	-4.04	0.74	1.36	8.89	850,402
Italy	58,943,464	76	94.17	-2.52	0.81	1.24	8.10	775,572
Spain	49,128,297	61	78.49	-2.43	0.78	1.29	8.42	805,382
Poland	36,497,495	53	58.31	-0.74	0.91	1.10	7.20	688,632
Romania	19,043,151	33	30.43	0.36	1.08	0.92	6.03	577,065
Netherlands	18,044,027	31	28.83	0.30	1.08	0.93	6.08	582,065
Belgium	11,883,495	22	18.99	0.42	1.16	0.86	5.64	540,159
Czechia	10,909,500	21	17.43	0.50	1.20	0.83	5.43	519,500
Portugal	10,749,635	21	17.17	0.53	1.22	0.82	5.35	511,887
Sweden	10,587,710	21	16.92	0.57	1.24	0.81	5.27	504,177
Greece	10,372,335	21	16.57	0.62	1.27	0.79	5.16	493,921
Hungary	9,539,502	21	15.24	0.80	1.38	0.73	4.75	454,262
Austria	9,197,213	20	14.69	0.74	1.36	0.73	4.80	459,861
Bulgaria	6,437,360	17	10.28	0.93	1.65	0.61	3.96	378,668
Denmark	5,992,734	15	9.57	0.75	1.57	0.64	4.17	399,516
Finland	5,635,971	15	9.00	0.83	1.67	0.60	3.93	375,731
Ireland	5,440,278	14	8.69	0.74	1.61	0.62	4.06	388,591
Slovakia	5,419,451	15	8.66	0.88	1.73	0.58	3.78	361,297
Croatia	3,874,350	12	6.19	0.81	1.94	0.52	3.37	322,863
Lithuania	2,890,664	11	4.62	0.89	2.38	0.42	2.75	262,788
Slovenia	2,130,850	9	3.40	0.78	2.64	0.38	2.47	236,761
Latvia	1,860,565	9	2.97	0.84	3.03	0.33	2.16	206,729
Estonia	1,369,995	7	2.19	0.67	3.20	0.31	2.04	195,714
Cyprus	982,966	6	1.57	0.62	3.82	0.26	1.71	163,828
Luxembourg	681,973	6	1.09	0.68	5.51	0.18	1.19	113,662
Malta	574,250	6	0.92	0.71	6.54	0.15	1.00	95,708
EU-27	450,646,971	720	720	0.00	1.00	1.00	6.54	625,899
Min	574,250	6	0.92	-5.21	0.72	0.15	1.00	95,708
Max	83,577,140	96	133.53	0.93	6.54	1.39	9.10	870,595

The unequal distribution of territorial power in the European Parliament (2025) is summarized in Figure 1, which shows the graph of electoral premiums $(s_i \forall i)$, and in Figure 2, which corresponds to the relative territorial efforts $(e_i \forall i)$ and the population representation indices $(r_i \forall i)$ of the EU-27 member states (2025).

The above results show that the European Parliament's electoral system does not comply with the principle "one person, one vote" (Balinski and Young, 2001), since, in a perfect proportional distribution, the values of relative territorial efforts e_i $(\forall i)$ and population representation indices r_i $(\forall i)$ must all be equal to one: $e_i = r_i = 1$ $(\forall i)$ -see Figure 2-.

Observation-1: The partial territorial representativeness indices, IP1 to IP6, can be adapted to the set of political forces competing in the European Parliament elections by replacing the total population P and the population vector $\vec{p} = (p_1, \ldots, p_n)$ with V and $\vec{v} = (v_1, \ldots, v_n)$, respectively, where V symbolizes the total number of votes taken into account in the electoral contest and v_i $(\forall i = 1,.,n)$ represents the number of votes obtained by the political force $i \in I$, I being, in this case, the set of participating political forces entitled to a seat.

Figure 1. Territorial electoral bonus (%) of the EU-27 Member States (10th Legislature: 2024-2029). Population 2025 (Eurostat, 2026).

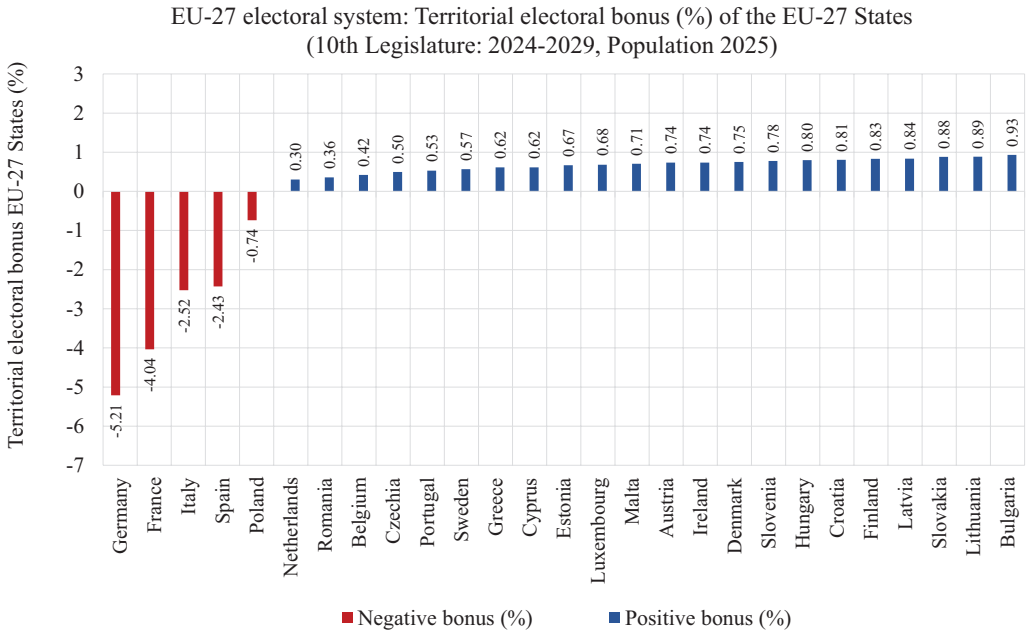
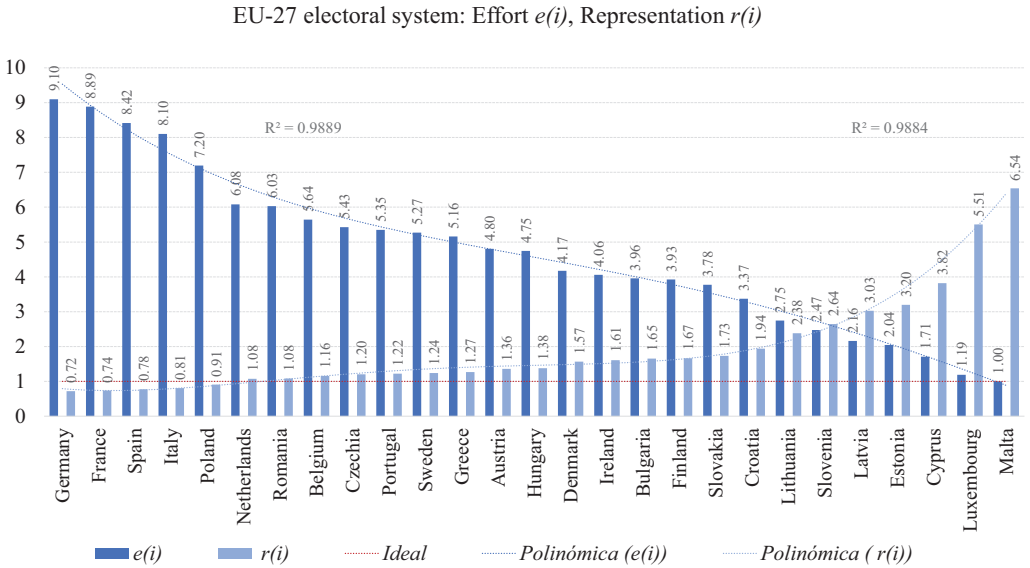


Figure 2. Relative territorial effort (e_i) and population representation index (r_i) of the EU-27 European Union states (10th Legislature: 2024-2029). Population 2025 (Eurostat, 2026).



3. Global metrics of territorial power disproportionality

Malapportionment is used to indicate a clear discrepancy between the percentage of seats allocated to each constituency and the percentages of the corresponding populations (Samuels and Snyder, 2001; Ansolabehere et al., 2003; Simón, 2009).

Malapportionment in a house of representatives can be measured in various ways, and there is no academic consensus on which index best measures a non-proportional allocation (Ocaña Lara and Oñate Rubalcaba, 2024; Urdánoz Ganuza, 2024).

Similar to Bautista (2025a, 2025b), eight disproportionality metrics (IG1 to IG8) are used to measure the unequal distribution of power among territories, based on two criteria:

a. Global indices that aggregate the differences between the proportion of seats allocated to each state and the corresponding proportion of its population.
b. Global indices that measure the maximum value of the partial indices.

IG1. *Loosemore-Hanby Index of disproportionality (1971)*

This corresponds to half the sum of the absolute values of the electoral bonuses for the set of states (Loosemore and Hanby, 1971), that is:

$$I_{LH} = \frac{1}{2}\sum_{i=1}^{n} |s_i| = \frac{1}{2}\sum_{i=1}^{n} \left|\xi_i - \pi_i\right| = \frac{1}{2}\sum_{i=1}^{n} \left|\frac{x_i}{H} - \frac{p_i}{P}\right| \quad (7)$$

The I_{LH} index represents the proportion of seats that have not been allocated strictly proportionally to the set of states.

The I_{LH} index is also half the rectangular distance (n-dimensional space) between the actual point $\vec{\xi} = (\xi_1, \ldots, \xi_n)$ of proportions of seats allocated to states and the ideal point $\vec{\pi} = (\pi_1, \ldots, \pi_n)$ of proportions of their populations.

IG2. *Rae Index of disproportionality (1971)*

It corresponds to the arithmetic mean of the absolute values of the electoral bonuses of the set of states (Rae, 1971), that is:

$$I_{Rae} = \frac{1}{n}\sum_{i=1}^{n} \left|s_i\right| = \frac{1}{n}\sum_{i=1}^{n} \left|\xi_i - \pi_i\right| = \frac{1}{n}\sum_{i=1}^{n} \left|\frac{x_i}{H} - \frac{p_i}{P}\right| \quad (8)$$

The I_{Rae} index is the arithmetic mean of the absolute values of electoral bonuses, including both overrepresentation and underrepresentation of states.

The I_{Rae} index is also the arithmetic mean of the absolute differences, in an n-dimensional space, between the actual point $\vec{\xi} = (\xi_1, \ldots, \xi_n)$ and the ideal point $\vec{\pi} = (\pi_1, \ldots, \pi_n)$.

IG3. *Gallagher Index of disproportionality (1991, 1992)*

It corresponds to the square root of half the sum of the squares of the electoral bonuses of the set of states (Gallagher, 1991, 1992), that is:

$$I_{Gal} = \sqrt{\frac{1}{2}\sum_{i=1}^{n} s_i^2} = \sqrt{\frac{1}{2}\sum_{i=1}^{n} (\xi_i - \pi_i)^2} = \sqrt{\frac{1}{2}\sum_{i=1}^{n} \left(\frac{x_i}{H} - \frac{p_i}{P}\right)^2} \quad (9)$$

The I_{Gal} index is also the Euclidean distance (n-dimensional space) between the actual point $\vec{\xi} = (\xi_1, \ldots, \xi_n)$ and the ideal point $\vec{\xi} = (\xi_1, \ldots, \xi_n)$ divided by $\sqrt{2}$.

IG4. *Sainte-Laguë Index of disproportionality (1910)*

This corresponds to the sum of the electoral bonuses of the states, squared and divided by the corresponding population proportions (Sainte-Laguë, 1910), that is:

$$I_{SL} = \sum_{i=1}^{n} \frac{s_i^2}{\pi_i} \equiv \sum_{i=1}^{n} \frac{(\xi_i - \pi_i)^2}{\pi_i} \quad (10)$$

IG5. *Disproportionality index of maximum deviation*

This corresponds to the maximum of the absolute values of the electoral bonuses for the set of states, that is:

$$I_{|s|_{max}} = \max_{i \in I}\left\{\left|s_i\right|\right\} = \max_{i \in I}\left\{\left|\xi_i - \pi_i\right|\right\} = \max_{i \in I}\left\{\left|\frac{x_i}{H} - \frac{p_i}{P}\right|\right\} \quad (11)$$

The index $I_{|s|_{max}}$ is the maximum value between the best electoral bonus and the worst electoral bonus with the sign changed.

The index $I_{|s|_{max}}$ is also the maximum absolute difference between the coordinates of the actual point $\vec{\xi} = (\xi_1, \ldots, \xi_n)$ and those of the ideal point $\vec{\pi} = (\pi_1, \ldots, \pi_n)$.

IG6. *Disproportionality indices of absolute and relative maximum electoral cost*

The disproportionality index of the maximum absolute electoral cost (IG0) corresponds to the maximum value of the inhabitants per seat ratio of the set of states, that is:

$$I_{c_{max}} = \max_{i \in I}\left\{c_i\right\} = \max_{i \in I}\left\{\frac{p_i}{x_i}\right\} \quad (12)$$

The index $I_{c_{max}}$ symbolizes the territorial electoral cost of the state with the least representation.

Meanwhile, the disproportionality index of the maximum relative electoral cost (IG6) is calculated by dividing $I_{c_{max}}$ by the average electoral cost $\overline{c}$ of the territory (EU-27):

$$I_{cr_{max}} = \frac{I_{c_{max}}}{\overline{c}} = \max_{i \in I}\left\{\frac{c_i}{\overline{c}}\right\} = \max_{i \in I}\left\{\frac{q_i}{x_i}\right\} = \max_{i \in I}\left\{cr_i\right\} \quad (13)$$

The index $I_{cr_{max}}$ is the relative cost, with respect to the average cost, of the worst-represented state.

IG7. *Disproportionality index of maximum ratio of advantage*

This corresponds to the maximum advantage ratio (population representation index) of the set of states, that is:

$$I_{r_{max}} = \max_{i \in I}\{r_i\} = \max_{i \in I}\left\{\frac{\xi_i}{\pi_i}\right\} = \max_{i \in I}\left\{\frac{x_i}{q_i}\right\} \quad (14)$$

The index $I_{r_{max}}$ is the advantage ratio of the most over-represented state.

IG8. *Disproportionality index of relative maximum electoral effort*

This corresponds to the maximum relative territorial effort of the set of states, that is:

$$I_{e_{max}} = \max_{i \in I}\{e_i\} = \max_{i \in I}\left\{\frac{c_i}{c_{\min}}\right\} : c_{\min} = \min_{i' \in I} c_{i'} \quad (15)$$

The index $I_{e_{max}}$ represents the relative territorial effort of the most underrepresented state (Bautista, 2025a, 2025b).

As a summary, Table 3 shows the value of the 8 global territorial indices (IG1 to IG8) and the absolute maximum electoral cost index ($I_{c_{max}}$) in the EU-27 European Parliament of the 10th Legislature (2024-2029), for the populations of the years 2024 and 2025.

Observation-2: The global metrics of territorial disproportionality, IG1 to IG8, are adaptable to the set of political forces by replacing the total population P with the total number of votes V in an election, and the population vector $\vec{p} = (p_1, \ldots, p_n)$ with the vector of votes obtained by the political forces $\vec{v} = (v_1, \ldots, v_n)$.

4. Models and methods of proportional distribution

4.1. Optimization models for the apportionment problem

The Apportionment Problem has numerous applications in the field of Industrial Engineering (Bautista et al., 1996, 2001; Bautista, 2020, 2021). However, it is in the political sphere where the first formalized contributions to this problem took place at the end of the 18th century in the emerging United States of America, when its leaders decided to determine the allocation of seats to each member state, with the idea that the number of inhabitants per seat should be as similar as possible in all states: the principle of "one person, one vote" (Balinski and Young, 2001).

In the abstract, the problem can be formalized as follows: given an integer H and a set I of n elements with an assignment of positive values, q_i $(i = 1, \ldots, n)$, called quotas, which satisfy $\sum_{\forall i} q_i = H$, the problem of proportional distribution consists of finding a set of n integer and non-negative values, x_i $(i = 1, \ldots, n)$, such that $\sum_{\forall i} x_i = H$ and that are as similar as possible to their corresponding quotas q_i (i.e. $x_i \cong q_i \ \forall i \in I$).

In practice, the integer value H usually corresponds to the number of available units of a scarce resource, the set I, of n elements, represents the recipients of the resource, and the quota values q_i are determined proportionally to positive attributes associated with the elements of set I.

In the political sphere, a system of representation is considered proportional if the number of seats x_i assigned to each constituency, or to each political force, is adjusted to its corresponding power quota q_i, that is: $x_i \cong q_i \ \forall i \in I$. These power quotas, if the principle of equity is taken into account, are determined from the population $\vec{p} = (p_1, \ldots, p_n)$ of each territory, or the votes $\vec{v} = (v_1, \ldots, v_n)$ obtained by each political force.

Under such conditions, the apportionment problem can be interpreted as an optimization problem, as formulated in the following mathematical program.

Pm-rp:

$$\min \ \mathcal{F}(\cdot) = \mathcal{F}(\vec{x}, \vec{q}, H) \quad (16)$$

subject to:

$$\sum_{i=1}^{n} x_i = H \quad (17)$$

$$x_i \in Z^+ \cup \{0\}, \quad \forall i = 1, \ldots, n \quad (18)$$

Where $\mathcal{F}(\cdot)$ is the objective function that measures a distance between the ideal proportional allocation point $\vec{q}$ and an actual allocation point $\vec{x}$. Equality (17) forces the allocation of all House (H) seats among constituencies (states). And constraints (18) establish the non-negative integrity of the variables x_i (seats).

4.2. Methods of allocating seats in houses of representatives

A method for allocating seats, whether proportional or not, is a mathematical formula—in reality, a well-defined algorithm or deterministic procedure—whose purpose is to

Table 3. Indices of territorial disproportionality. European Parliament (tenth Parliament: 2024-2029).

Year	I_{LH}%	I_{Rae}%	I_{Gal}%	I_{SL}%	$I_{\|s\|_{max}}$%	$I_{cr_{max}}$	$I_{r_{max}}$	$I_{e_{max}}$	$I_{c_{max}}$
2024	14.96	1.11	5.79	22.81	5.23	1.39	6.65	9.26	869,334
2025	14.94	1.11	5.79	22.67	5.21	1.39	6.54	9.10	870,595

transform the population of the constituencies (e.g., states, provinces, etc.) or the votes of the political parties into seats. The methods of allocating seats form the basis of electoral systems.

This paper applies various methods of seat allocation in the European Parliament of the EU-27, taking into account the conditions of its Tenth Legislature (2024-2029).

MR0. *European Electoral System EU-27 (2024)*

In each legislative term, the composition of the European Parliament is determined by the European Council. In the last elections (June 2024), the allocation of the 720 seats in the current European Parliament among its member states was carried out in accordance with Decision (EU) 2023/2061, resulting in the values shown in Table 1 (European Council, 2023).

The European Electoral System establishes (Article 1) a series of restrictions that do not favour a proportional distribution based on the population quotas of its Member States; among them:

1. The System allocates a minimum of 6 seats per Member State, regardless of its population.
2. The System limits the maximum number of seats a Member State can obtain in the European Parliament to 96.
3. The System introduces and applies the principle of "decreasing proportionality," which means that, given two Member States, i and j, if the population of the first State is greater than that of the second ($p_i > p_j$), then the absolute territorial electoral cost of the first State must be greater than that of the second ($c_i > c_j$).

Figure 3 shows the allocation of seats in the EU-27 electoral system versus seat quotas.

MR1. *Hamilton's method*

One of the most widely used classic proportional allocation procedures is the method of Alexander Hamilton (1755-1804), also known as the method of largest remainders, which was first proposed in 1792 in the then fledgling United States of America. This method is implemented using the A0 algorithm.

Algorithm A0: Hamilton's method (Seat allocation among states)

Step 1:	A quota *(Q)* is obtained by dividing the total population *(P)* by the number of seats to be distributed *(H)*; that is:
Step 2:	The quota of seats for each State ($q_i \forall i$) is determined by dividing the population of the State ($p_i \forall i$) by the allocation quota *(Q)*; that is: $q_i = p_i / Q \; \forall i \in I$.
Step 3:	Each State is allocated as many Deputies ($x_i^0 \forall i$) as the integer part of its quotas ($q_i \forall i$); that is: $x_i^0 = \lfloor q_i \rfloor \; \forall i \in I$.
Step 4:	The remaining Deputies ($R = H - \sum_{\forall i} x_i^0$) are assigned one by one to the states that have the largest decimal fractions: $\varphi_i = q_i - x_i^0 \; \forall i \in I$.
Step 5:	Following the previous allocation, the final distribution of seats is $x_i \; \forall i \in I$. End

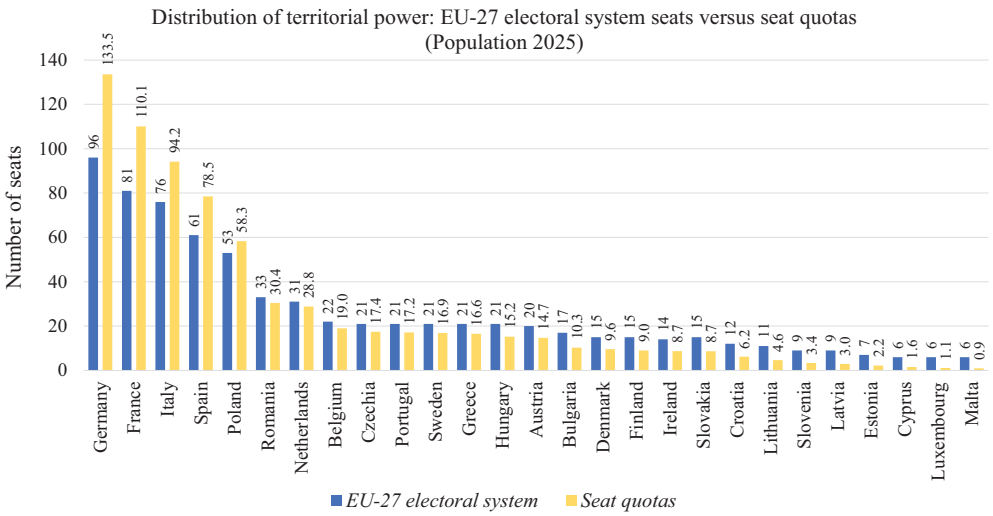
An interesting property satisfied by some proportional allocation procedures is the quota property (Balinski and Young, 1975), whose definition is as follows:

An integer distribution solution $\vec{x} = (x_1, \ldots, x_n)$ is said to satisfy the quota property with respect to the quotas of seats or ideal values $\vec{q} = (q_1, \ldots, q_n)$ if the following holds:

$$\lfloor q_i \rfloor \leq x_i \leq \lceil q_i \rceil \; \forall i = (1, \ldots, n) \tag{19}$$

where $\lfloor q_i \rfloor$ is the largest integer less than or equal to q_i (Floor function) and $\lceil q_i \rceil$ is the smallest integer greater than or equal to q_i (Ceiling function).

Figure 3. Allocation of seats according to the electoral system of the European Parliament (EU-27) versus the values of the seat quotas (10th Legislature: 2024-2029. Population 2025).



If $x_i \geq \lfloor q_i \rfloor \ \forall i$, then $\vec{x}$ is said to satisfy the lower quota property.

If $x_i \leq \lceil q_i \rceil \ \forall i$, then $\vec{x}$ is said to satisfy the upper quota property.

The solutions offered by Hamilton's method satisfy the quota property due to the way they are constructed (see algorithm A0).

Figure 4 compares the current distribution of seats in the European Parliament (European Council, 2023) with the solution offered by Hamilton's method.

Hamilton's method minimizes the sum of absolute differences, $|x_i - q_i| \ (\forall i)$, raised to any power $p \geq 1$. Particularly, Hamilton's method minimizes the sum of absolute differences ($p = 1$) and the sum of quadratic differences ($p = 2$). In general, Hamilton's method minimizes the Minkowski distance of order $p \geq 1$ between the distribution point $\vec{x} = (x_1, \ldots, x_n)$ and the quota point $\vec{q} = (q_1, \ldots, q_n)$; that is:

$$\min \mathcal{F}_H(\cdot) = \sum_{i=1}^{n} \left| x_i - q_i \right|^p ; \ \min \mathcal{F}_H(\cdot) = \left(\sum_{i=1}^{n} \left| x_i - q_i \right|^p \right)^{\frac{1}{p}} (p \geq 1) \tag{20}$$

MR2. *Jefferson-D'Hondt method*

A second classic method for allocating seats, used in many states, is the well-known D'Hondt method (D'Hondt, 1878), developed by Victor D'Hondt (1841-1901), which is equivalent to the method proposed in 1792 by Thomas Jefferson (1743-1826). Jefferson's method can be used to allocate seats both between states and between political parties, the latter being its more common application.

One way to apply Jefferson's method is to find a partitioning divisor D_J, identical for all EU-27 states, such that the sum of the integer parts of the quotients between the populations ($p_i \ \forall i$) of each state and D_J is equal to the number of seats to be allocated H (for the EU-27: $n = 27$, $H = 720$), that is:

$$Find \ D_J : \sum_{i=1}^{n} \left\lfloor \frac{p_i}{D_J} \right\rfloor = \left\lfloor \frac{p_1}{D_J} \right\rfloor + \cdots + \left\lfloor \frac{p_n}{D_J} \right\rfloor = H \tag{21}$$

Jefferson's method disadvantages minorities, since for a state to obtain its first seat it must have a population no less than the minimum value of the divisor D_J that satisfies (21).

Jefferson's method satisfies the lower quota property, since its solutions meet the condition: $x_i \geq \lfloor q_i \rfloor \ \forall_i$.

Figure 5 compares the current distribution of seats in the European Parliament with the solution offered by the Jefferson-D'Hondt method.

Another way to apply the Jefferson-D'Hondt method is to divide the populations $p_i(\forall i)$ or the quotas $q_i(\forall i)$ by the series of integers $\{1,2, \ldots, H\}$. The resulting quotients are then ordered from highest to lowest, and the seats are assigned, one by one, to the states or political parties, selecting the H largest quotients.

In terms of optimization, the Jefferson method minimizes the maximum advantage ratio and also maximizes the minimum relative electoral cost, among other functions; that is:

Figure 4. Allocation of seats according to the electoral system of the European Parliament (EU-27) versus the values of Hamilton's method (10th Legislature: 2024-2029. Population 2025).

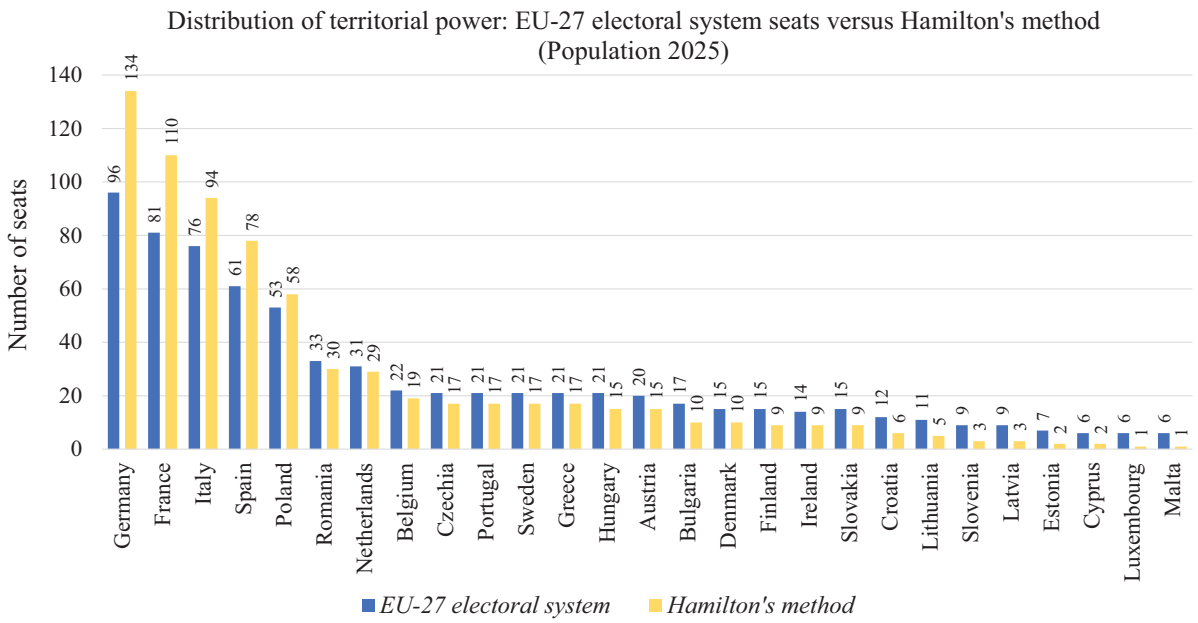
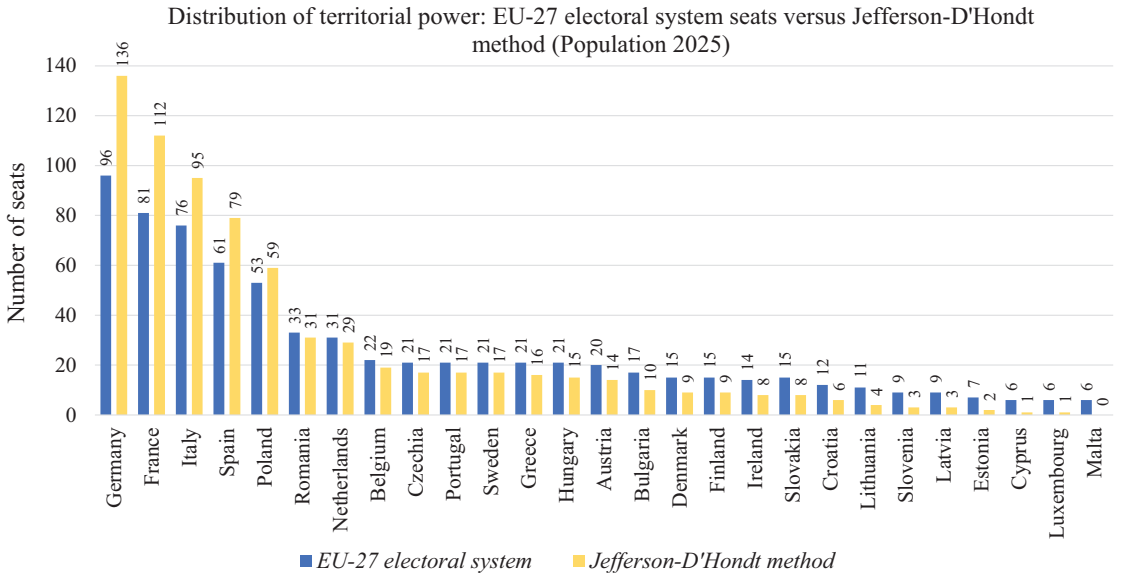


Figure 5. Allocation of seats according to the electoral system of the European Parliament (EU-27) versus the values of Jefferson-D'Hondt method (10th Legislature: 2024-2029. Population 2025).



$$\min \mathcal{F}_J(\cdot) = \max_{i\in I}\{r_i\} = \max_{i\in I}\left\{\frac{x_i}{q_i}\right\} \Leftrightarrow$$
$$\max \mathcal{G}_J(\cdot) = \min_{i\in I}\{cr_i\} = \min_{i\in I}\left\{\frac{q_i}{x_i}\right\} \qquad (22)$$

MR3. *Adams's method*

A third, less frequently used, classic procedure for allocating seats is the method of John Quincy Adams (1767-1848), which dates back to 1832.

The original application of Adams' method (1832) consists of finding a divisor D_A, the same for all states, such that the sum of the quotients between the populations $(p_i\ \forall i)$ of each state and D_A, applying the ceiling function to the quotients, is equal to the number of seats to be allocated H, that is:

$$\text{Find } D_A : \sum_{i=1}^{n}\left\lceil\frac{p_i}{D_A}\right\rceil = \left\lceil\frac{p_1}{D_A}\right\rceil + \cdots + \left\lceil\frac{p_n}{D_A}\right\rceil = H \qquad (23)$$

The current application of the Adams' method, or the method of small divisors, consists of dividing the quotas q_i $(\forall i)$ by the series of integers {0,1, ..., H–1}, ordering the resulting quotients from largest to smallest, and allocating the seats, one by one, to the constituencies or political parties, selecting the H largest quotients.

The Adams' method benefits minorities, as it assigns a first seat (division by zero) to every constituency (state) or every political party regardless of the number of inhabitants or the number of votes, provided that H is greater than or equal to n.

Adams' method satisfies the upper-quota property, as its solutions meet the condition: $x_i \le \lceil q_i \rceil\ \forall i$.

Figure 6 compares the current distribution of seats in the European Parliament with the solution provided by Adams' method.

Considering the optimization, Adams' method minimizes the maximum absolute electoral cost c_{max} and the maximum relative electoral cost cr_{max}, and also maximizes the minimum advantage ratio r_{min}; that is:

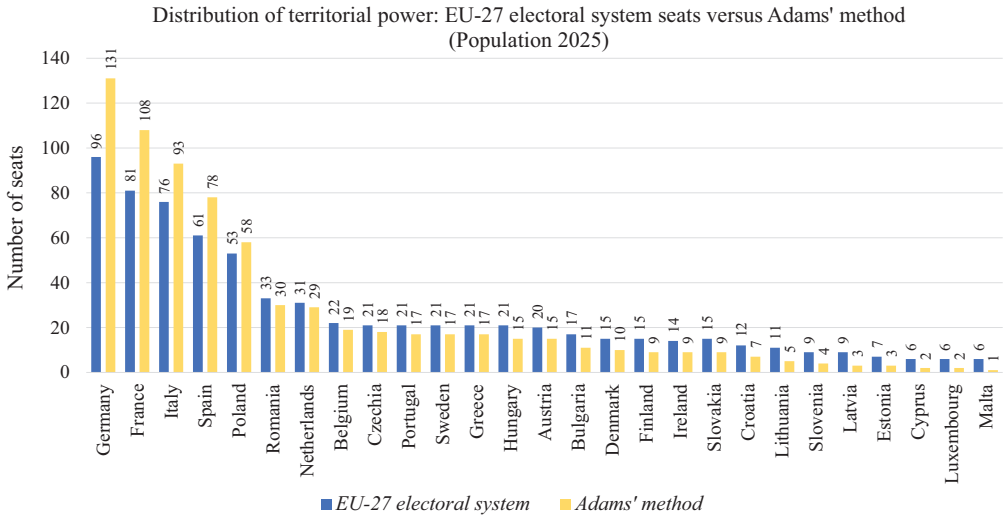
$$\min \mathcal{F}_A(\cdot) = \max_{i\in I}\{cr_i\} = \max_{i\in I}\left\{\frac{q_i}{x_i}\right\} \Leftrightarrow$$
$$\max \mathcal{G}_A(\cdot) = \min_{i\in I}\{r_i\} = \min_{i\in I}\left\{\frac{x_i}{q_i}\right\} \qquad (24)$$

MR4. *Webster's method*

A fourth classic divisor procedure in line with those of Jefferson and Adams is the method of Daniel Webster (1782-1852), which dates back to 1832, and which is known in Europe as the method of André Sainte-Laguë (1882-1950).

The classic application of Webster's method (1832) consists of determining a divisor D_W, the same for all constituencies (states), such that the sum of the quotients between the populations $(p_i\ \forall i)$ of the states and D_W, rounding the quotients to integer values, is equal to the number of seats to be distributed H, that is:

Figure 6. Allocation of seats according to the electoral system of the European Parliament (EU-27) versus the values of Adams' method (10th Legislature: 2024-2029. Population 2025).



$$Find\ D_W : \sum_{i=1}^{n} \left\lfloor \frac{p_i}{D_W} \right\rfloor = \left\lfloor \frac{p_1}{D_W} \right\rfloor + \cdots + \left\lfloor \frac{p_n}{D_W} \right\rfloor = H \tag{25}$$

The current application of Webster's method consists of dividing the quotas $q_i(\forall i)$ by the series of odd numbers {1,3,5, …, $2H-1$}, ordering the resulting quotients from highest to lowest and assigning the seats, one by one, to the constituencies or political forces, selecting the H largest quotients.

Observation-3: Dividing the quotas $q_i(\forall i)$ by the series {1, 3, 5, …, $2H-1$} or by the sequence {0.5; 1.5; 2.5; …; $H-0.5$}, leads to the same solution for seat distribution; this fact allows us to define Webster's series, in its genuine form, as the arithmetic mean sequence of Adams' and Jefferson's series.

Webster's method minimizes the sum of quotients between the squared differences $(x_i - q_i)^2$ $(\forall i)$ and the corresponding quotas $q_i(\forall i)$: that is:

$$\min \mathcal{F}_W(\cdot) = \sum_{i=1}^{n} \frac{(x_i - q_i)^2}{q_i} \tag{26}$$

The distribution of seats provided by Webster's method in the European Parliament coincides with that offered by Hamilton's method (Figure 4).

MR5. *Hill-Huntington method*

The fifth classical method of distribution is that of Joseph Hill (1911), or the Hill-Huntington method, which consists of dividing the quotas $q_i(\forall i)$ by the geometric mean sequence of the Adams and Jefferson series: {0; 1.41; 2.45; … ; $\sqrt{(H(H+1)}$}. The Hill method optimizes function (27), among others.

$$\min \mathcal{F}_{HH}(\cdot) = \sum_{i=1}^{n} \frac{(x_i - q_i)^2}{x_i} \tag{27}$$

The distribution of seats provided by the Hill-Huntington method in the European Parliament coincides with that of the Hamilton and Sainte-Laguë methods (Figure 4).

MR6. *Dean's method*

The sixth classical method of seat allocation is that of James Dean (1832), which locally optimizes function (28) using the harmonic mean sequence of the Adams and Jefferson series, namely: {0; 1.33; 2.40; …; $(H(H+1)/(H+0.5)$}.

$$\min \mathcal{F}_D(\cdot) = \max_{\{i,j\} \subseteq I} \left| \frac{q_i}{x_i} - \frac{q_j}{x_j} \right|, \text{ local optimization: } 2-opt \tag{28}$$

The solution of Dean's method also coincides, in this case, with the solution of Hamilton's method (Figure 4).

4.3. Traditional Huntington's methods and divisor methods

The methods of Adams, Dean, Hill, Webster, and Jefferson constitute the five traditional methods of apportionment of Edward Huntington (1874-1952).

Huntington's traditional methods (Huntington, 1921) are applied by fixing an initial feasible allocation of seats $\vec{x}: \sum_{\forall i} x_i = H$ and transferring seats, one by one, between every pair (i,j) of elements of I as long as perfect proportionality improves, which can be represented in several ways. For example, the perfect proportionality associated with the representation indices takes the form: $x_i/q_i = x_j/q_j \ \forall(i,j)$. Then, if $i \in I$ is overrepresented with respect to the element $j \in I$, that is: $x_i/q_i > x_j/q_j$, the indices δ_1 and δ_2 are determined as follows:

$$\delta_1 = \left|\frac{x_i}{q_i} - \frac{x_j}{q_j}\right|; \delta_2 = \left|\frac{x_i - 1}{q_i} - \frac{x_j + 1}{q_j}\right| \tag{29}$$

Thus, if $\delta_2 < \delta_1$, a seat is transferred from $i \in I$ to $j \in I$, and this process continues until no improvement is achieved by transferring seats between pairs of elements in I. Huntington's methods are particular cases of the class known as Divisor Methods. Table 4 shows the characteristics of several divisor methods that are used or have been proposed in electoral systems.

The divisor methods are a type of allocation procedure whose results depend on the divisor used. There are infinitely many methods of divisors (Balinski and Young, 2001), including Huntington's five traditional methods, the Belgian method, the Sainte-Laguë method, and the modified Sainte-Laguë method (see Table 4).

Each divisor method is associated with a divisor criterion. A divisor criterion is a real function $d(x)$ defined on the non-negative integers $x = 0,1,...,H$.

A divisor criterion is considered proportional, or more precisely, to provide a proportional distribution, when it satisfies two properties:

p1. Increasing monotonicity: $d(x) < d(x+1) \ \forall x$.

p2. Membership in intervals with integer endpoints: $d(x) \in [x, x+1], \forall x = 0, 1, \ldots, H$.

where x symbolizes the number of seats allocated to a generic constituency (State).

The divisor methods with divisors criteria that satisfy **p1** and **p2** follow the principle "one person, one vote", interpreting that the quotients between quotas q_i $(\forall i)$ and corresponding seats x_i $(\forall i)$ are close to unity: $q_i/x_i \cong 1$ $(\forall i)$.

Observation-4: The divisor of the Belgian method, $(d(x) = (x+2)/2$, does not meet the interval membership condition: $d(x) \in [x, x+1] \ \forall x$, starting from the allocation of the third seat $(x \geq 3)$, therefore, such a method cannot be considered a proportional distribution method.

The approximation of quotas to seats, $q_i/x_i \cong 1$ $(\forall i)$, can be achieved by minimizing the maximum value of the quotients $q_i/d(x_i)$ $(\forall i)$, giving rise to a minimax optimization problem with integer variables, so that the generic objective function $\mathcal{F}(\cdot)$ of formula (16) of the **Pm-rp** model can be specified as follows:

$$min \mathcal{F}(\cdot) = \max_{i \in I} \left\{ \frac{q_i}{d(x_i)} \right\} : d(x) \in [x, x+1] \wedge$$
$$d(x) < d(x+1), \ \forall x \, \forall i \in I \tag{30}$$

This simplification allows for the definition of infinite proportional distribution methods that can be applied using the same algorithm, let's say A1.

Algorithm A1: Divisor Methods (Allocation of seats among territories)

Step 0:	Define divisor criterion $d(x)$: $d(x) < d(x+1)$, $d(x) \in [x, x+1] \ \forall x$.
Step 1:	Do $x_i = 0 \ \forall i \in I$. Reset the seat allocation counter: $k = 0$.
Step 2:	Calculate the quotas of the member states: $q_i = H(p_i/P)$ $\forall i \in I$.
Step 3:	Calculate the quotients: $\delta_i = q_i / d(x_i) \ \forall i \in I$. If $d(x_i) = 0$, Do $\delta_i \to \infty$(For example: $\delta_i = 10^{12}$).
Step 4:	Determine the State i^* with the highest quotient δ_i: $i^* = argmax_{i \in I}\{\delta_i\}$.
Step 5:	Assign a seat to State i^*: Do $x_{i^*} \leftarrow x_{i^*} + 1$. Update the counter: $k \leftarrow k + 1$.
Step 6:	Completion test: If $k \leq H$, go to Step 3; otherwise, finish.

Note that algorithm A1 can be adapted to the distribution of power between political forces by replacing the total population P with the total number of votes V, and the territorial quotas $H(p_i/P)$ $\forall i$ with the quotas of the political forces $H(v_i/V)$ $\forall i$.

Table 4. Attributes of various practical divisor methods in electoral systems.

Name	$d(x_i)$: $x_i = 0, 1, \ldots, H$	*Series of divisors*	*State*
Adams	x_i	0, 1, 2, 3, 4, …	USA
Dean	$x_i(x_i+1)/(x_i+0.5)$	0, 1.33, 2.40, 3.43, …	USA
Hill-Huntington	$\sqrt{x_i(x_i+1)}$	0, 1.41, 2.45, 3.46, …	USA
Webster	$x_i + 0.5$	0.5, 1.5, 2.5, 3.5 …	USA
Jefferson-D'Hondt	$x_i + 1$	1, 2, 3, 4, 5, …	ES, FR
Sainte-Laguë	$2x_i + 1$	1, 3, 5, 7, 9 …	DE, SE
Sainte-Laguë^M	1.4, …, $(2x_i + 1)$	1.4, 3, 5, 7, 9 …	DE, SE
Belgian	$(x_i + 2)/2$	1, 1.5, 2, 2.5, 3 …	BE

5. Methods and indices in the European parliament: 10th legislature

5.1. Territorial distribution of seats in the European Parliament

In summary, Table 5 shows the results of applying the Hamilton, Adams, Dean, Hill, Webster, Jefferson and Belgian methods to the European Parliament in the 10th Legislature, with the EU-27 population of 2025 (Eurostat, 2026).

Based on Table 5, the following conclusions can be drawn:

1. The Hamilton, Dean, Hill, and Webster (Sainte-Laguë) methods all offer the same solution for allocating seats among the EU-27 member states based on the 2025 population.
2. The Adams method disadvantages Germany, France, and Italy compared to the Hamilton method.
3. The Jefferson-D'Hondt method benefits Germany, France, Italy, Spain, Poland, and Romania compared to the Hamilton method.
4. The Jefferson-D'Hondt method leaves Malta without representation.
5. The Belgian method leaves Cyprus, Luxembourg, and Malta without representation.

5.2. Indices of territorial disproportionality in the European Parliament

The fairness or unfairness of an electoral system when applied to a set of territories or political forces can be measured in many ways.

To measure unfairness, it is reasonable to establish metrics that assess the differences or ratios between the electoral costs of territories that exert more effort to obtain a

Table 5. Allocation of seats by various allocation methods in the European Parliament (10th Legislature: 2024-2029) with the 2025 population census (Eurostat, 2026).

State	*EU-27*	q_i	*Ham.*	*Adams*	*Dean*	*Hill*	*Webster*	*Jeff.*	*Belgian*
Germany	96	133.53	134	131	134	134	134	136	139
France	81	110.05	110	108	110	110	110	112	114
Italy	76	94.17	94	93	94	94	94	95	98
Spain	61	78.49	78	78	78	78	78	79	81
Poland	53	58.31	58	58	58	58	58	59	60
Romania	33	30.43	30	30	30	30	30	31	31
Netherlands	31	28.83	29	29	29	29	29	29	29
Belgium	22	18.99	19	19	19	19	19	19	18
Czechia	21	17.43	17	18	17	17	17	17	17
Portugal	21	17.17	17	17	17	17	17	17	17
Sweden	21	16.92	17	17	17	17	17	17	16
Greece	21	16.57	17	17	17	17	17	16	16
Hungary	21	15.24	15	15	15	15	15	15	15
Austria	20	14.69	15	15	15	15	15	14	14
Bulgaria	17	10.28	10	11	10	10	10	10	9
Denmark	15	9.57	10	10	10	10	10	9	9
Finland	15	9.00	9	9	9	9	9	9	8
Ireland	14	8.69	9	9	9	9	9	8	8
Slovakia	15	8.66	9	9	9	9	9	8	8
Croatia	12	6.19	6	7	6	6	6	6	5
Lithuania	11	4.62	5	5	5	5	5	4	3
Slovenia	9	3.40	3	4	3	3	3	3	2
Latvia	9	2.97	3	3	3	3	3	3	2
Estonia	7	2.19	2	3	2	2	2	2	1
Cyprus	6	1.57	2	2	2	2	2	1	0
Luxembourg	6	1.09	1	2	1	1	1	1	0
Malta	6	0.92	1	1	1	1	1	0	0
UE-27	720	720	720	720	720	720	720	720	720

seat and those of territories that exert less effort to achieve the same result.

As an example of cost ratios, Figure 7 compares the current relative territorial efforts of the EU-27 member states in 2025 with those provided by the Hamilton method. In any case, the fairest method is the one that minimizes the unfairness metric defined by the decision-maker.

Assuming that injustice and disproportionality of an allocation method are synonymous, any global metric of territorial power disproportionality can be valid for our purpose. Therefore, we will evaluate the global disproportionality indices, IG1 to IG8, applying various allocation methods in the European Parliament.

Table 6 presents the results for the global indices IG1 to IG8 after applying the allocation methods discussed here.

Figure 8 shows the graphical results of the absolute indices IG1 to IG5, depending on the methods, grouped by index type.

Based on Table 6 and Figure 8, we can draw the following conclusions about the values of the Absolute Indices:

1. The electoral system currently in place in the EU-27 (10th Legislature: 2024-2029. Census 2025) is undoubtedly the worst method of seat allocation among those analyzed, when considering the five absolute indices of disproportionality (IG1 to IG5). For example, the Gallagher index

Figure 7. Comparison of the relative territorial effort e_i ($\forall i$) of the EU-27 Member States in the European Parliament: EU-27 Electoral System versus Hamilton's method (10th Legislature: 2024-2029. Population 2025). The Hamilton, Dean, Hill and Webster solutions show the same values.

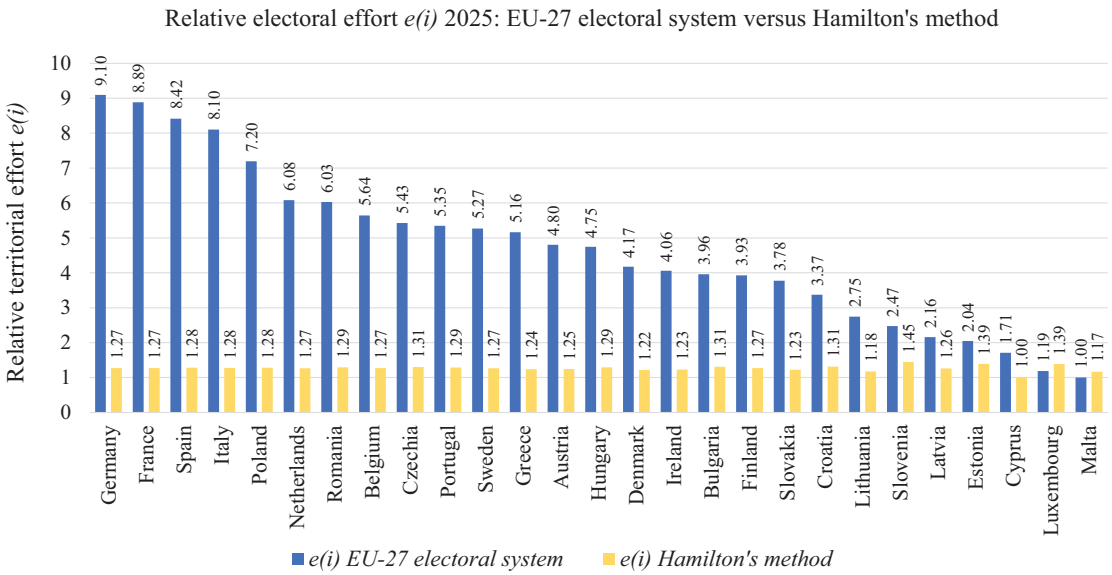
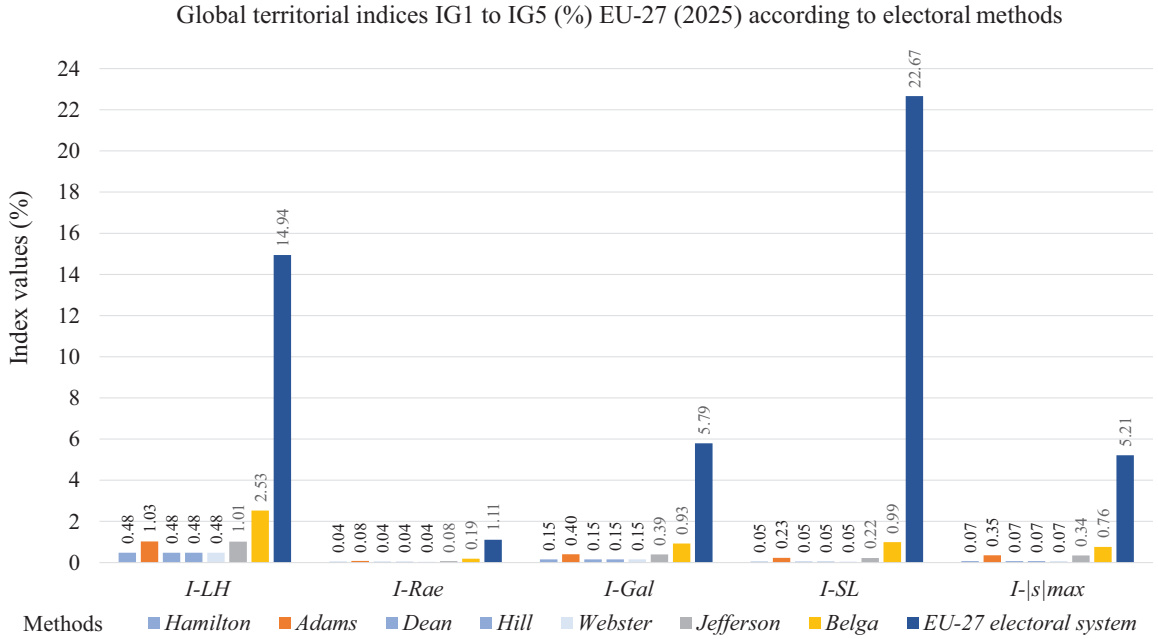


Table 6. Values of the global territorial disproportionality indices, IG1 to IG8, according to the seat allocation methods applied to the European Parliament (10th Legislature: 2024-2029. Population 2025). The Hamilton, Dean, Hill, and Webster solutions show the same values.

	Absolute Indices					Relative Indices			Cost
Method\IG#	I_{LH}%	I_{Rae}%	I_{Gal}%	I_{SL}%	$I_{\|s\|_{max}}$%	$I_{cr_{max}}$	$I_{r_{max}}$	$I_{e_{max}}$	$I_{c_{max}}$
Hamilton	**0.48**	**0.04**	**0.15**	**0.05**	**0.07**	1.13	1.27	**1.45**	710,283
Adams	1.03	0.08	0.40	0.23	0.35	**1.02**	1.84	1.87	**637,993**
Dean	**0.48**	**0.04**	**0.15**	**0.05**	**0.07**	1.13	1.27	**1.45**	710,283
Hill	**0.48**	**0.04**	**0.15**	**0.05**	**0.07**	1.13	1.27	**1.45**	710,283
Webster	**0.48**	**0.04**	**0.15**	**0.05**	**0.07**	1.13	1.27	**1.45**	710,283
Jefferson	1.01	0.08	0.39	0.22	0.34	1.57	**1.02**	1.60	982,966
Belgian	2.53	0.19	0.93	0.99	0.76	2.19	1.04	2.28	1,369,995
UE-27 E.S.	14.94	1.11	5.79	22.67	5.21	1.39	6.54	9.10	870,595
Min	0.48	0.04	0.15	0.05	0.07	1.02	1.02	1.45	637,993
Max	14.94	1.11	5.79	22.67	5.21	2.19	6.54	9.10	1,369,995

Figure 8. Global indices of territorial disproportionality, IG1 to IG5, applying seat allocation methods to the European Parliament (10th Legislature: 2024-2029. Population 2025). Grouping by index type.



value in the European Parliament is 5.79% a very high value compared to Hamilton's 0.15% or Adams' 0.40%.

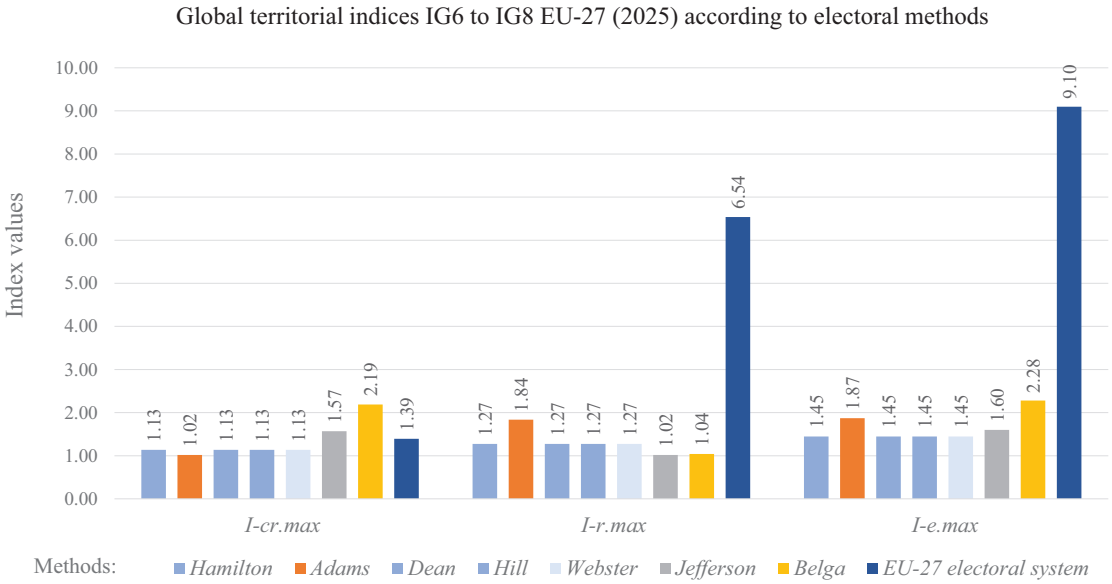
2. The Loosemore-Hanby index value associated with the EU-27 electoral system (Population 2025) is 14.94%. This value is very high, as it implies that almost 15% of the seats in the European Parliament (108 seats) are located outside the territory to which they should be allocated proportionally. It also means that almost 15% of EU-27 citizens (67.3 million) are not represented through their own Member State.
3. The Belgian method stands out as the worst allocation method after the one currently used in the European Parliament, according to the five absolute indices (IG1 to IG5).
4. The Jefferson's method is more proportional than the Adams' method according to the five absolute indices, and less proportional, according to these indices, than Hamilton, Dean, Hill, and Webster.
5. Hamilton, Dean, Hill, and Webster are the most proportional methods here; in fact, the seat-allocation solution they provide (Table 5) is optimal for all five absolute indices (Table 6). This is because Hamilton minimizes the indices I_{LH}, I_{Rae}, I_{Gal} and $I_{|s|_{max}}$, and Webster minimizes the index I_{SL}. Furthermore, this solution satisfies the quota property by matching Hamilton's solution.

Figure 9 shows the graphical results of the relative indices IG6 to IG8, according to the allocation methods, grouped by index type.

From Table 6 and Figure 9, we can draw the following conclusions:

1. The index of maximum relative electoral cost $I_{cr_{max}}$ (and absolute $I_{c_{max}}$) does not discriminate with regard to the proportionality of the methods. According to this index, the current electoral system in the EU-27 is more proportional than the Jefferson's method. Based on these indices, the best method is the Adams' method, because it minimizes both the maximum absolute and relative electoral cost.
2. The value of the maximum advantage ratio index associated with the EU-27 electoral system is very high: $I_{r_{max}} = 6.54$, since one state (Malta) is 6.5 times more represented in seats than it would be entitled to based on its population quota. According to this index, the best method is the Jefferson's method, as it minimizes the maximum advantage ratio.
3. The value of the index of maximum relative electoral effort associated with the EU-27 electoral system is very high: $I_{e_{max}} = 9.10$, since one state (Germany) must exert more than nine times the effort to win a seat than the state that exerts the least effort to achieve the same (Malta). Colloquially, the electoral power of one Maltese citizen is equivalent to that of nine German citizens. According to this index, the best method is Hamilton's, along with those of Dean, Hill, and Webster, in this case.

Figure 9. Global indices of territorial disproportionality, IG6 to IG8, applying seat allocation methods to the European Parliament (10th Legislature: 2024-2029. Population 2025). Grouping by index type.



6. Conclusions

The following general conclusions can be drawn from this study:

1. The electoral system that determines the composition of the European Parliament, favoring the representation of the EU-27 member States, cannot be classified as proportional from the perspective of the scientific community dedicated to resolving the so-called apportionment problem.
2. The electoral system that determines the composition of the European Parliament does not comply with the principle of "one person, one vote".
3. To measure malapportionment, various global metrics are used that aggregate the differences between the proportion of seats allocated to territories and the corresponding population proportion, or that determine the maximum value of a set of partial indices.
4. Every global metric of malapportionment has an associated allocation method that minimizes it; therefore, once the metric is chosen, it is sufficient to apply the method that offers an optimal solution. This fact transforms the proportional allocation problem into an optimization problem.
5. No single overall metric is inherently better than another, although the metrics referred to here as absolute indices are more informative because they take into account all participants in the electoral contest.
6. No single allocation method minimizes all overall metrics.
7. When drafting an electoral law, it is more practical to define the index (or indices) to be optimized rather than debating which allocation method is better or worse.

And, as specific conclusions from this analysis, we have:

1. The most proportional methods are Hamilton, Dean, Hill, and Webster, all of which offer the same distribution solution. This solution satisfies the quota property and, furthermore, minimizes the five absolute indices: I_{LH}, I_{Rae}, I_{Gal}, I_{SL} y $I_{ls|_{max}}$.
2. The least proportional method corresponds to the electoral system currently in place in the EU-27, when all five absolute indices are considered.
3. The second least proportional method is the Belgian method when all five absolute indices are considered.
4. The least proportional method corresponds to the electoral system currently in force in the EU-27 when the maximum advantage ratio index $I_{r_{max}}$ and the maximum relative effort index $I_{e_{max}}$ are considered.
5. The least proportional method corresponds to the Belgian method when the maximum relative electoral cost index $I_{cr_{max}}$ or the maximum absolute electoral cost index $I_{c_{max}}$ is taken into account.

As for future lines of work, it is proposed to use the methodology described here to analyze the disproportionality of the distribution of power in other territories and in electoral contests in which political forces participate.

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